Teaching Experience and Teaching Statement

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Of all the classes I've taught, the one which best illustrates my teaching philosophy is the Undergraduate Mathematics Laboratory. I designed it with Peter Sarnak as part of Princeton's VIGRE grant to foster more collaboration between undergraduates, graduate students, postdocs and faculty. I ran the course four times there, once each with Peter Sarnak, Andrew Wiles, Ramin Takloo-Bighash, and Yakov Sinai. This course allowed juniors to investigate current unsolved conjectures theoretically and numerically. I have taught variants of this course at the Courant Institute, NYU, with Peter Sarnak, a similar course (primarily for graduate students) with Vitaly Bergelson and Warren Sinnott at The Ohio State University, and four advanced undergraduate research class at Brown (one with Jill Pipher). I have run a summer version twice, by myself in 2004 at Ohio State and in 2003 at AIM with Brian Conrey, David Farmer, Chris Hughes and Mike Rubinstein.

I love both mathematical research and teaching, and the research classes and programs are an ideal way to combine the two. While we use computers and programs in the class, they are used as a tool to interest students in mathematics; very quickly, students are able to see phenomena no one has seen before, and we have found this sense of discovery and ownership greatly increases students' interest and motivation. While such classes take a lot of time, they offer a real chance at making a difference in someone's mathematical career, and allow me to explore a variety of topics. Students see the problems and techniques used in current research early in their studies, and are better prepared to decide their career path. While these programs have led to several joint papers with undergraduates (from graph theory to random matrix theory to elliptic curves), it is important to note that this is not the goal. The purpose is to choose interesting problems which are accessible to beginning students; over the years I have successfully supervised undergraduates ranging from entering freshmen knowing only BC calculus to advanced undergraduates preparing for graduate study.

I wrote a textbook based on these courses, *An Invitation to Modern Number Theory* (with Ramin Takloo-Bighash). The book is similar to the courses: the goal is to introduce enough background material for students to see the mathematical landscape. Topics include elementary number theory, Diophantine equations, continued fractions, probabilistic number theory, equidistribution, and L-functions and random matrix theory. Because of my extensive experience supervising undergraduate and graduate students, as well as serving on three dissertation committees (one in analysis and two in number theory), I feel ready to advise graduate students on thesis problems.

I have also taught numerous standard courses. In these classes I always provide advanced material and open research problems. Students respond to a challenge, and enjoy the opportunity to further explore the subject. Mathematics has been successfully applied to many problems in the real world, and these connections can be used to keep the students interested and motivated. Examples from my classes include cryptography (Abstract Algebra), scheduling problems for airlines (Linear Algebra), and determining why certain baseball statistics predict a team's performance (Mathematical Statistics). While detailed descriptions of all the courses are provided later, the two courses I taught last semester highlight my teaching philosophy and style, and are briefly summarized below.

I have taught Math 162 (Mathematical Statistics) for the past three years at Brown, and will teach it again next spring. This is a theory intensive statistics course. In addition to proving standard results such as the central limit theorem (assuming a standard result from complex analysis), we introduce numerous useful proof techniques and explore their applications. For example, we discuss differentiating identities and matching coefficients and show applications to formulas for tests based on runs. The course is peppered with numerous applied problems with interesting theoretical components (many of the examples come from student projects, a required and popular component of the class). Examples range from analysis of baseball games (where the fact that games cannot end in ties lead us to the concept of structural zeros in $r \times c$ tables) to Benford's law and digit bias (which is related to important problems in equidistribution theory, and is currently being used by the IRS to detect corporate tax fraud).

The other course is Math 54 (Honors Linear Algebra). This is another theory-intensive course; however, in addition to covering the theory we also discussed the computational aspects. Linear programming was originally a theoretical oddity; several early papers said the results are interesting, but impractical as one will never be able to compute the desired quantities in a reasonable amount of time. We used discussions of the computational problems (calculating eigenvalues, propagation of errors in solving equations, efficiently multiplying matrices) as springboards to the theory. Advanced topics included linear programming, fast multiplication, and random matrix theory.

The following are the last two years of my teaching ratings (taking from the Critical Review, an undergraduate run evaluation of teaching at Brown); 1 is high and 5 is low. Two of my courses were the highest rated, and a third was the second highest (and the highest among classes with at least 10 students).

Semester	Class	Rating	Math Dept Range	Mean
Spring 07	Mathematical Statistics	1.15	[1.07, 2.21]	1.58
Spring 07	Honors Linear Algebra	1.39	[1.07, 2.21]	1.58
Fall 06	Abstract Algebra	1.28	[1.28, 2.91]	1.85
Spring 06	Mathematical Statistics	1.22	[1.22, 2.01]	1.54
Spring 06	Freshman Seminar	1.60	[1.22, 2.01]	1.54
Fall 05	Abstract Algebra	1.47	[1.28, 2.66]	1.61

Finally, I have been the faculty advisor to the undergraduate math clubs at Ohio State and Brown University. Duties range from giving lectures to helping them organize student conferences to running Putnam sessions to teaching them how to use LaTeX and programming environments such as Mathematica and Matlab to advising them on how to write papers and give talks (for example, this year 5 of my undergraduates will give talks at national research conferences).

1 VIGRE Undergraduate Mathematics Laboratory

1.1 Purpose

For the past seven years I have been involved in the design and implementation of a new advanced undergraduate class at Princeton, NYU, and Ohio State. Sponsored by a VIGRE grant from the NSF, Professor Sarnak and I have created an Undergraduate Mathematics Laboratory at Princeton. We then brought the class to the Courant Institute, NYU, and I then brought the course to Ohio State. I have run a smaller version at Brown (by myself and with Jill Pipher in 2006).

The statements and simple cases of many interesting conjectures are accessible to undergraduates; further, there is often very little numerical support for the sweeping generalizations that are claimed in the conjectures. Undergraduates learn the necessary theory and quickly become involved in cutting edge research. They see what types of problems mathematicians study, and experience what it would be like to be a graduate student by doing original (guided) research.

While many graduate students and most postdocs have taught a standard class, few have mentored students or helped design research programs; additionally, many graduate students have yet to conduct original research, write papers or give research talks. Under the supervision of the faculty, these skills are built by having the graduate students help choose problems to investigate with the undergraduates they are mentoring. The integrated nature of the program provides valuable training and exposure not seen in a typical class or research group.

1.2 Course Structure

The courses run roughly as follows: the professors and postdocs lecture on various topics for a few weeks. Participants often come from very diverse backgrounds with different skills, and supplemental lectures are given on needed background material. For example, to investigate certain conjectures we rarely need an entire course on Probability or Complex Analysis, but rather a few key results. Students are quickly given problems to think about / experiment with; as the semester progresses, the students and staff break into smaller groups. The general lectures are replaced with more specific presentations. At the end of the semester, students submit a .tex version of their work, documented computer codes, and present their research to the class.

When possible, we try to choose an integrated series of problems, so that students will be able to converse with each other about their work. We have worked hard to maintain good faculty to student ratios (usually about 1:3 or 1:4). For example, at Ohio State we had two senior faculty, two post-docs, and eight undergraduate and graduate students. One group was constructing families of elliptic curves with moderate to high rank, which was used by another group looking at the effect of additional zeros at the central point on the first zero above the central point, both of which help a third group investigating family dependent lower order corrections to distribution of zeros near the central point.

Typically these classes have a few faculty members, several graduate students or postdocs, and an undergraduate computer assistant. These are mathematics classes, not computer classes. While many undergraduates have programming experience, not much is required to do good investigations, due to the power of packages such as Mathematica, Maple, Matlab, Magma, C/PARI, etc. Not all the staff needs to be expert in these systems; typically only a subset is, and an undergraduate with strong skills is sometimes hired to be point man for debugging. Also, after running these courses for years, we have assembled an extensive database of documented programs, which makes it easy for future investigators with little computer experience to continue these researches.

1.3 Topics and Results

Projects from previous years are available on-line:

- Princeton: http://www.math.princeton.edu/∼mathlab/index.html
- NYU: http://www.williams.edu/go/math/sjmiller/public_html/math/generalmath/uml@nyu/index.html
- AIM (summer program): http://www.williams.edu/go/math/sjmiller/public_html/math/generalmath/AIM/
- Ohio State (summer program): http://www.williams.edu/go/math/sjmiller/public_html/math/generalmath/

Some previous topics of investigation include

1. Hardy-Littlewood Circle Method (Varieties; Goldbach; Germain Primes)
2. Random Matrix Theory (band matrices; Toeplitz and Palindromic Toeplitz matrices; truncated Cauchy matrices; sparse matrices)

3. Ramanujan Graphs / Random Graphs / k-Regular Graphs
4. Elliptic Curves (Birch and Swinnerton-Dyer Conjecture; excess rank, points of low height, signs of functional equations, first zero above the central point, spacings between zeros; Sato-Tate, constructing one-parameter families with rank)
5. Primality Testing
6. Equidistribution of Roots of Polynomials mod p
7. Continued Fractions (distribution of digits; special families; closed form expressions; periodic continued fractions)
8. Poissonian Behavior (especially of $\{n^k\alpha\}$)
9. Dynamical Piston
10. Lone Runner
11. Interval Exchanges
12. $3x + 1$ Problem
13. Benford's Law and Digit Bias

1.4 My Duties

My duties range from designing the class and choosing problems to giving background and advanced lectures to supervising undergraduate and graduate research and finally to mentoring graduate students and postdocs in how to advise and mentor undergraduates and design research programs. I am also responsible for teaching the participants how to use the software (C, PARI, Matlab, Maple, Mathematica, LaTeX), as well as providing help with programming and efficient algorithm design.

1.5 Summary

Designing and running the UML has shown me what it is like to supervise undergraduate and graduate research. What matters most is not the background of the participants, but their enthusiasm and interest in mathematics. There is no dearth of accessible problems, and in fact some of the best projects were done by students with the weakest background. With other professors, I've supervised a variety of simultaneous research projects (eight students the first year, eleven the second, twenty in the third, eight in the fourth and three in the fifth); I've also helped supervise 30 students in the last four summers, and many students (individually and in small groups) at Brown. The UML has given me two rare opportunities: I'm the organizer / coordinator for a research group, and I've been able to participate in the design and creation of a non-standard class. I've also learned how to budget my time so that my own research does not suffer.

It has been a very exciting and enjoyable class to teach. I'm helping students investigate unsolved conjectures, and helping to train / work with graduate students and postdocs. While it is time consuming, the problems are exciting and worth doing, and at the end of the year I have helped prove new results (or at least gather experimental evidence in their support). It does take significantly longer to solve and write-up a project than it would if I worked alone, but I am exposed to a lot more mathematics and I help train future colleagues, benefiting from their enthusiasm and viewpoints.

Results from these researches have been presented at conferences and accepted in research journals for publication. Also, Ramin Takloo-Bighash and I have written a book, *An Invitation to Modern Number Theory* (Princeton University Press, 2006). The purpose of our book is to provide an introduction to current problems in Number Theory. While capable of being used as a standard text, it is based on lectures, problems and results from the past few years of the UML. Several new results in the fields are contained, as well as extensive literature review and lists of additional topics for research.

Finally, the course has evolved to more heavily include graduate students. Building on our success with undergraduates, we are expanding the role of graduate students. Instead of solely involving upper level graduate students as mentors, we now have pre-Generals graduate students, who can explore an area of mathematics before having to choose a thesis topic. Both sets of graduate students are given assistance in mentoring and learning how to lecture on current research.

2 Standard Classes

I've taught the following subjects (detailed descriptions below):

1. Linear algebra: graduate linear algebra, honors linear algebra, linear algebra.
2. Calculus: honors multivariable calculus, calculus I.
3. Statistics: mathematical statistics.
4. Algebra: abstract algebra.
5. Discrete mathematics: discrete mathematical structures.
6. Seminars: freshmen seminar (from riddles to modern mathematics), reading classics working group (on Euler), fractal geometry working group.
7. Problem solving: honors problem solving, advanced honors problem solving.
8. Pre-calculus: fundamentals of college algebra.

2.1 Brown University

2004: Math 9 (Introductory Calculus - one of five sections, fall semester).

2004: Math 52 (Linear Algebra – course coordinator, fall semester).

2005, 2006: Math 900-3B: How to think the Calculus way (summer). This is a first semester calculus course for high school students. Teaching beginner's calculus (especially to students with poor mathematics backgrounds) was very different from the other classes I've done. Each time I had an extremely bimodal class. Half the students could not do basic algebra, while the other half had already seen much of calculus. It was a real challenge designing the lectures and problem sets for such an audience.

2005, 2006: Math 153 (Abstract Algebra, fall semester). This is the only advanced course required for undergraduate math majors at Brown. The class is typically bimodal, and the material and course must be structured to fit the needs of each group. Applications include number theory and cryptography.

2005, 2006, 2007, 2008: Math 162 (Mathematical Statistics, spring semester).

2006: Math 1 (Freshmen Seminar: From riddles to modern mathematics). This seminar is a chance to show students what types of problems mathematicians study, and how math can be applied.

Topics included origami, Rubik's cubes, soap bubbles, planetary motion, cryptography, Babylonian mathematics, combinatorics, Benford's Law and detecting tax fraud, and voting theory.

2007: Math 54 (Honors Linear Algebra, spring semester). Topics include efficiency of computation, linear programming, and applications to statistics and random matrix theory.

2007: Math 35 (Honors Calculus, fall semester).

2.2 The Ohio State University

2003: Discrete Mathematical Structures (Math 566), summer quarter.

2003: Basic College Mathematics (Math 104), autumn quarter.

2003: Honors Problem Solving and Advanced Honors Problem Solving (Math 187, 487), autumn quarter: introduction to modern mathematics through problem solving, for freshman and sophomores, and juniors and seniors. In addition to the class, I ran weekly Putnam problem sessions.

2003: Working Group on Fractal Geometry (with Gerald Edgar and Larry Lindsay), autumn.

2003: Working Group on Euler (with Vitaly Bergelson and Warren Sinnott), autumn and winter quarters.

2004: Graduate Linear Algebra (Math 683L), summer quarter. Topics included linear programming and Random Matrix Theory, and introduced first year students to current research topics.

2004: The Circle Method and Diversions through Number Theory, one week course for the Ross Program (talented high school and college students), summer.

2.3 Princeton

1996 – 1999: I was the administrator / coordinator for the Math Department's review sessions for all classes from Introductory Calculus (Math 101) to Honors Linear Algebra (Math 204).

1997 – 1998: I was the Head Teaching Assistant / Class Coordinator for Honors Multivariable Calculus (Math 203) and Honors Linear Algebra (Math 204), and taught a section of Math 204.

1999: Upon requests from many of my former students, I was asked to TA for Discrete Mathematics (Computer Science 341).

1999, 2001, 2002: I taught Introduction to Calculus (Math 101) at Princeton's Freshman Scholars Institute. FSI is a summer program for incoming freshmen who have weak mathematical backgrounds, yet want to pursue majors in the sciences or engineering. Classes are five to seven students, meeting every day (for six weeks) for an hour and a half.