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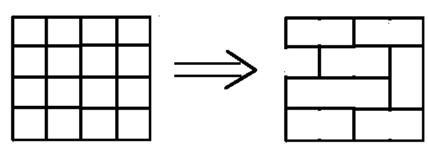
Thanksgiving Conundrum

Theorem 0.1. *Suppose we have a four-by-four array of turkey cages such that adjacent cages share a side. The minimum number of sides that can be removed to break the perimeter of all squares formed (including* 1×1*,* 2×2*,* 3×3*, and* 4×4*) is nine sides.*

Proof. First we will show that we must remove at least nine sides. Since the outer perimeter (which is a 4×4 square) must be broken, an outer perimeter piece must be removed. This will break the perimeter of one of the sixteen 1×1 squares, leaving behind fifteen. Note that removing a side can break the perimeter of at most two 1×1 squares, since no side belongs to more than two such squares. As fifteen more of these squares must have their perimeters broken, we will have to remove at least eight more sides. Thus nine is a lower bound for the number of sides that must be removed.

To see that removing nine sides will suffice, consider the following diagram:



Note that nine sides have been removed, and no square's perimeter has been left intact. We have therefore achieved our lower bound, and conclude that nine is the minimum number of sides that can be removed to attain the desired result.

(In the figure above, only one turkey would escape. Lucky bird.) □